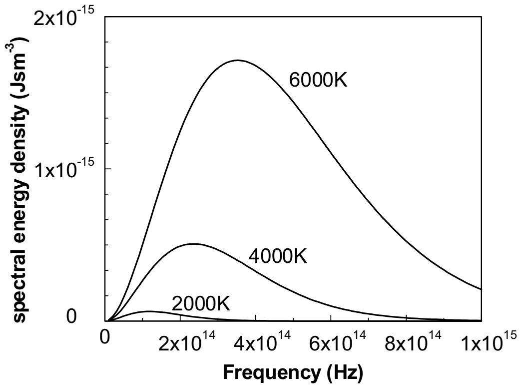
Appendix A: Black Body Radiation

The emission of radiation from hot bodies was extensively studied in the 19^{th} century using the laws of thermodynamics, and then consequently by quantum theory. It was the unsolved problem of explaining the spectrum from hot bodies that led Planck to propose that energy was quantized in 1901.

All hot objects emit radiation and also absorb radiation emitted by the surrounding objects. In thermodynamic equilibrium, the amount of energy absorbed must exactly balance the amount of energy emitted, unless the object has an internal energy source that accounts for the difference e.g. the sun or a light bulb. Some objects absorb and emit better than others. “Black bodies” absorb everything that hits them by definition. Shiny objects are less good at absorbing (and emitting: see 1 below).

Experimental measurements show that the spectrum emitted by a hot object is determined only by the temperature *T*. This gives rise to the concept of black body radiation with the spectrum shown below:



Here is a summary of the classical laws of the black body radiation:

1. The ratio of the spectral emissive power to the spectral absorptivity for all bodies is a universal function of wavelength and temperature only. (Kirchoff’s Law).

2. The total power radiated is proportional to the fourth power of the temperature. (Stefan’s Law):

$$W = \sigma T^4 \qquad \text{(A.1)}$$

where W is the power emitted per unit area and σ is Stefan’s constant (5.67×10^{-8} $Jm^{-2}s^{-1}K^{-4}$).

3. The wavelength of the peak in the spectrum is inversely proportional to the inverse of the temperature (Wien’s displacement law):

$$\lambda_{\max} T = 2.9 \times 10^{-3} \text{ m K} \qquad \text{(A.2)}$$

Thermodynamics can explain these results, but cannot explain the detailed shape of the energy spectrum. This requires quantum theory. In 1901 Planck derived the following formula for the spectral energy density u(ν,T) of the black body radiation:

$$\mathrm{u}(\nu) = \frac{8\pi h \nu^3}{c^3} \, \frac{1}{\exp\left(h\nu / kT\right) - 1}. \qquad \text{(A.3)}$$

u(ν)dν is the energy per unit volume of black body radiation with frequency between ν and ν+dν. This result exactly matches the experimental data. Stefan’s and Wien’s laws can be derived from it. It was a great triumph for Planck, and heralded the dawn of quantum theory.

Reading

F. Mandl, *Statistical Physics*, chapter 10

E. Hecht, *Optics* 3^{rd} edition (1998), §13.1.1

R. Eisberg and R. Resnick, *Quantum Physics of atoms, molecules, solids, nuclei, and particles,* chap. 1